Power grid complex network evolutions for the smart grid

Pagani, Giuliano Andrea; Aiello, Marco

Physica A 396 (2014) 248–266

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Power grid complex network evolutions for the smart grid

Giuliano Andrea Pagani *, Marco Aiello

Distributed Systems Group, Johann Bernoulli Institute for Mathematics and Computer Science, University of Groningen, Groningen, The Netherlands

HIGHLIGHTS

- Network design for smart grid.
- Complex network for power grid design.
- Effects of topology on the cost of electricity distribution.
- Complex network literature topologies comparison for smart grid.

ARTICLE INFO

Article history:
Received 18 July 2013
Received in revised form 20 September 2013
Available online 16 November 2013

Keywords:
Complex network
Network evolution
Power grid
Smart grid

ABSTRACT

The shift towards an energy grid dominated by prosumers (consumers and producers of energy) will inevitably have repercussions on the electricity distribution infrastructure. Today the grid is a hierarchical one delivering energy from large scale facilities to end-users. Tomorrow it will be a capillary infrastructure at the medium and low voltage levels that will support local energy trading among prosumers. We investigate how different network topologies and growth models facilitate a more efficient and reliable network, and how they can facilitate the emergence of a decentralized electricity market. We show how connectivity plays an important role in improving the properties of reliability and path-cost reduction. Our results indicate that a specific type of evolution balances best the ratio between increased connectivity and costs to achieve the network growth.

1. Introduction

Something is changing in the way energy is both produced and distributed, due to the combined effects of technological advancements and the introduction of new policies. In the last decades a clear trend has invested the energy sector: that of *unbundling*. This is, the process of dismantling monopolistic and oligarchic energy system, by allowing a greater number of parties to operate in a certain role of the energy sector and market. The goal of unbundling is that of reducing costs for the end-users and providing better services through competition (e.g., Refs. [1,2]). From the technological perspective, new energy generation facilities (mainly based on renewable sources) are becoming more and more accessible. These are increasingly convenient and available at both the industrial and the residential scale [3]. The new actors operating in this scenario, who are both producers and consumers of energy, also known as *prosumers*, are increasing in number and will most likely demand a market with total freedom for energy trading [4]. In this future setting, the main role of the high voltage grid will change, leaving more space and relevance for the distribution grid (i.e., medium voltage and low voltage). In fact, the energy interactions between prosumers will increase and occur at a rather local level, therefore involving the low and medium voltage grids, inevitably calling for an upgrade of the enabling distribution infrastructure in order to facilitate local

* Corresponding author. Tel.: +31 503633984.
E-mail addresses: g.a.pagani@rug.nl (G.A. Pagani), m.aiello@rug.nl (M. Aiello).

0378-4371/$ – see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.physa.2013.11.022

energy exchanges. This vision for the infrastructure is comparable to a ''peer-to-peer'' system on the Internet, rather than the current strictly hierarchical system. But how will the infrastructure evolve or change to enable and follow this trend?

The starting point of our study is to assume that the infrastructure must change to accommodate the new way of producing and distributing energy [5]. The tool for our investigation is that of Complex Network Analysis (CNA) [6–9]. In particular, in the present case we use CNA as an engineering tool to synthesize networks using topological models coming from the literature of modeling the evolution of technological, infrastructural and social network. Our goal is to provide a methodology to support the change by statistically looking at how the current infrastructure should evolve and estimating the benefits of the evolutions while keeping an eye on the associated costs. In a nutshell, we intend to provide the foundations for a decision support system for high level planning the upgrade of the distribution network. We base our study on actual samples of the Dutch grid and previous results that provided an initial economic analysis of the possible barriers from an infrastructure point of view to delocalized trades [10]. The present paper considers growth models for network topologies providing an analysis of which models suit best the purpose of local energy exchange. In order to evaluate the adequacy of the generated networks, we develop a set of metrics, based on CNA literature and our own experience, that capture the various aspects that networks suited for small-scale energy exchange need to satisfy. It is then quite straightforward to compare the results of the synthetic models with the real samples and, on that ground, propose network models that best suit a prosumer-based local energy exchange. Finally, a quantitative evaluation of how the improvement in the topology directly influences electricity transport prices is then possible considering the metrics defined in the literature. In simple words, we look at the possible evolutions of the current grid that would make most sense to achieve the vision of a smart grid from the point of view of the prosumer. The study is statistical and can provide a budgeting and decision support tool for governments and utilities.

We remark the novelty of this proposal with respect to previous CNA studies of the power grid. In the survey work [11], it is emphasized that the use of CNA is mainly on the high voltage networks to get information on resilience to failures, while the medium and low voltage grids have been mostly neglected. Another novelty is the use of CNA not as a tool for pure analysis of the existing infrastructure, but to exploit it as an infrastructure design tool. Using Graph Theory in the design of distribution systems is not completely new, several studies have incorporated Graph Theory elements in operation research techniques for grid planning [12], but never, to the best of our knowledge, has Graph Theory been combined with global statistical measures to design the grid. In addition, we ground the design methods into investments by taking into account the costs of grid cabling based on the types of cables typically used in real distribution networks (i.e., Northern Netherlands medium and low voltage network samples). The exploitation of the advancement in network studies and topology provides a new way of looking at the development of the power grid infrastructure. A future grid that will have much of its production decentralized will call for an adequate infrastructure whose topology and development has to take advantage of the modern development of network models and network metrics to analyze its properties. In addition to the physical constraints dictated by the Kirchhoff's laws, the grid will have to obey a set of metric and constraints coming from the scientific approach at studying networks to have an efficient and optimized infrastructure. In summary, the paper discovers which topologies according to CNA-based metrics are best suited in terms of performance and reliability of the infrastructure for a local energy exchange, gives an estimation of the cabling cost for the realization of such topologies and assesses the advantages from the electricity distribution point of view of the proposed topologies compared to the current ones. From the point of view of Power Systems, we propose a new way to look at distribution grid planning. Our proposal is to consider statistical tools for estimating benefits and costs of upgrading the infrastructure. This can be a high-level decision-support tool in the hands of grid planners and governmental organizations. To the best of our knowledge, this type of approach to power grid planning, called for by the shift towards decentralized generation, is novel.

The paper—which is a condensed version of the on-line available and unpublished technical report [13] to which we refer for detailed simulation data—is organized as follows. Section 2 describes the main properties of the network models considered, while the metrics utilized to compare the properties of the various generated graphs are described in Section 3. The analysis and discussion of the results is presented in Section 4. The economic aspects of denser networks are evaluated in Section 5, while an overall discussion of the evolution of topologies is in Sections 6 and 7 reviews the related work, while concluding remarks are in Section 8. Appendix provides definitions of the metrics used in the evaluation of the models.

2. Modeling the power grid

We resort to complex network analysis, a branch of Graph Theory having its root in the early studies of Erdős and Rényi [14] on random graphs and considering statistical structural properties of very large graphs. CNA is a relatively young field of research. The first systematic studies appeared in the late 1990s [15,16] having the goal of looking at the properties of large networks with a complex systems behavior. Afterwards, CNA has been used in many different fields of knowledge, from biology [17] to chemistry, from linguistics to social sciences [18], from telephone call patterns [19] to computer networks [20] and the web, to virus spreading [21], logistics and also inter-banking systems [22]. Men-made infrastructures are particularly interesting to study under the CNA lenses, especially when they are large scale and grow in a decentralized and independent fashion, thus not being the result of a global, but rather of many local autonomous designs. The power grid is a prominent example. In this work, we consider a novel approach both in using CNA tools as a design instrument (i.e., CNA-related metrics are used in finding the most suited medium and low voltage grid for local energy exchange) and in focusing on the medium and low voltage layers of the power grid. In fact, traditionally, CNA studies

Table 1
Categories of Medium and low voltage network and their *order* based on Ref. [10].

Network layer	Category	Order
Low voltage	Small	≈20
Low voltage	Medium	≈90
Low voltage	Large	≈200
Medium voltage	Small	≈250
Medium voltage	Medium	≈500
Medium voltage	Large	≈1000

applied to the power grid only evaluate reliability issues and disruption behavior of the grid when nodes or edges of the high voltage layer are compromised, e.g., Ref. [23].

In particular, we study models for graph generation proposed for technological complex networks. For each model we evaluate the properties of the network for several values of the *order* of the graph. Following our analysis of the Northern Dutch medium and low voltage grids [10], we categorize networks as *Small*, *Medium* and *Large*, see Table 1. We then analyze the properties of the networks generated from by synthetic models by applying relevant CNA metrics and combining them appropriately. In this way, CNA is not only a tool for analysis, but it becomes a design tool for the future electrical grid.

Here we look at network models that have proven successful in showing salient characteristics of technological networks (i.e, preferential attachment, Copying Model, power-law networks), social networks (i.e., small-world, Kronecker graph, recursive matrix) and natural phenomena as well (e.g., Random Graph, small-world, Forest Fire) to investigate which one is best suited for supporting local-scale energy exchange from a topological point of view. Next we provide a brief introduction to all the models used in the present study, while a more in-depth presentation is available for instance in Ref. [24] or [25]. A graphical representation of the topological models generated is shown in Fig. 1.

Random Graph. A Random Graph is a graph built by picking each possible pair of nodes and connecting them with an edge with probability p. It was formalized in the pioneering studies of Erdős and Rényi [14].

Small-world Graph. The small-world phenomenon became known after the works of Milgram in the sociological context [18] who found short chains of acquaintances connecting random people in the USA. More recently, the small-world characterization of graphs has been investigated by Watts and Strogatz [26,15], who showed the presence of the small-world property in many types of networks such as actor acquaintances, the power grid and neural networks in worms.

Preferential Attachment. The preferential attachment model represents the phenomenon happening in real networks where a fraction of nodes has a high connectivity while the majority of nodes has small node degree. This model is built upon the observation by Barabási and Albert [16] of a typical pattern characterizing several type of natural and artificial networks.

R-MAT. R-MAT (Recursive MATrix) is a model that exploits the representation of a graph through its adjacency matrix [27]. In particular, it applies a recursive method to create the adjacency matrix of a graph, thus obtaining a self-similar graph structure. This model captures the community-based pattern appearing in some real networks.

Models independent from the average node degree

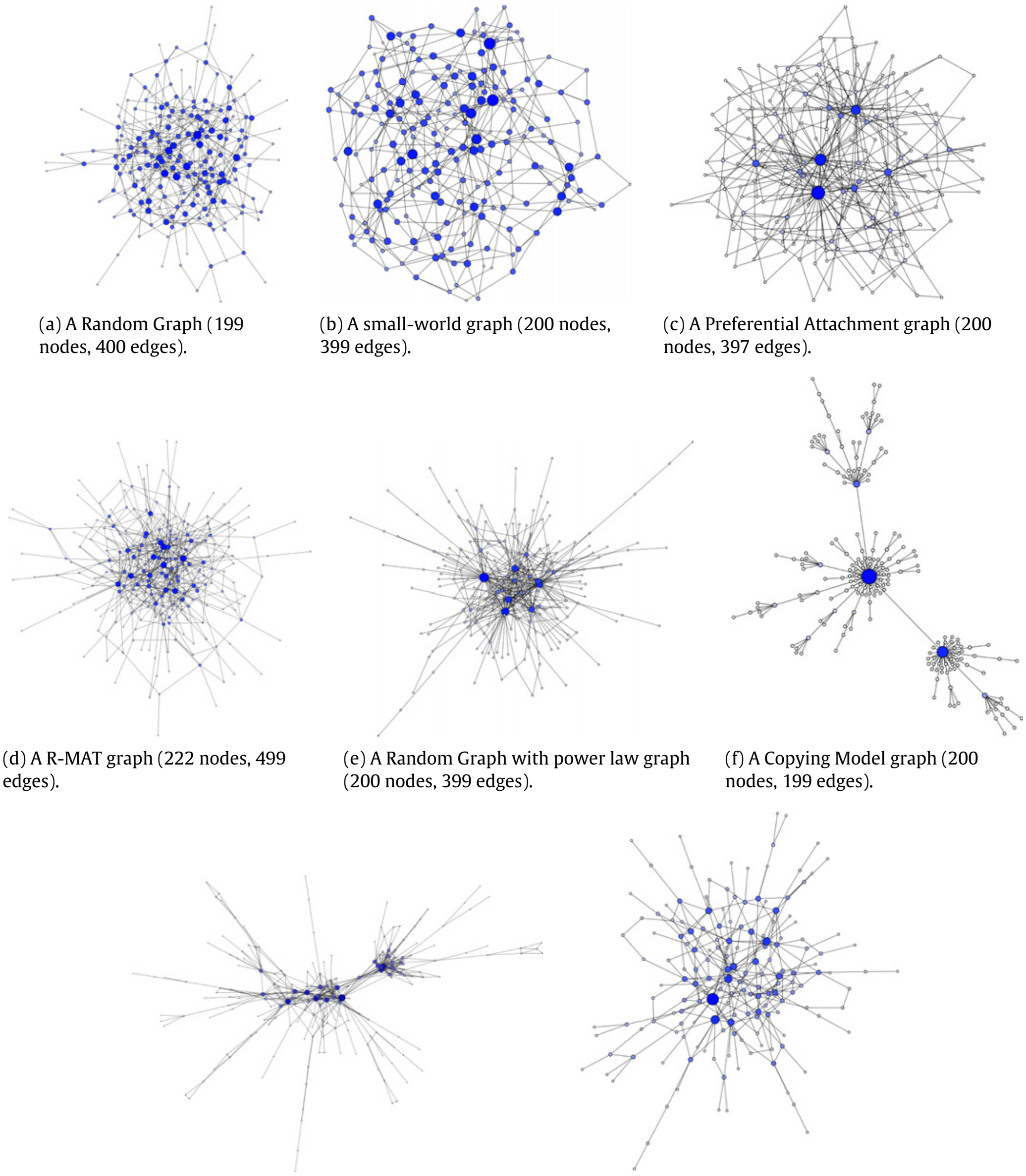
When generating certain models there is no explicit dependence on the average node degree, these include Random Graph with power-law model, Copying Model, Forest Fire and Kronecker Graph which are presented next. Some of these models produce networks with power-law node degree distribution. Power grids infrastructures do not follow power-laws in the degree distribution, but rather an exponential one [28]. However, in this design phase of a future grid we do not want to exclude a topology a priori.

Random Graph with Power-law. A Random Graph with power-law model generates networks characterized by a power-law in the node degree probability distribution ($P(k) \sim k^{-\gamma}$) having the majority of nodes with a low degree and a small number of nodes with a very high degree. Power-law distributions are very common in many real life networks both created by natural processes (e.g., food-webs, protein interactions) and by artificial ones (e.g., airline travel routes, Internet routing, telephone call graphs) [9].

Copying Model. Replicating the structure underlying the links of WWW pages brought the Copying Model [29] capturing the tendency of members of communities with same interests to create pages with a similar structure of links.

Forest Fire. In order to capture dynamic aspects of the evolution of networks, Leskovec et al. [30] proposed the Forest Fire model. The intuition is that networks tend to densify in connectivity and shrink in diameter (i.e., the greatest shortest path in the network) during the growth process. Technological, social and information networks show this phenomenon in their growth process.

Kronecker Graph. A generating model with a recursive flavor similar to R-MAT uses the Kronecker product applied to the adjacency matrix of a graph [31]. If the Kronecker product is applied to the same matrix, therefore multiplying the matrix with itself recursively, a self similar structure arises in the graph. This model creates networks that show a densification in the connectivity of its nodes, thus providing a shrinking diameter over time.



(a) A Random Graph (199 nodes, 400 edges).

(b) A small-world graph (200 nodes, 399 edges).

(c) A Preferential Attachment graph (200 nodes, 397 edges).

(d) A R-MAT graph (222 nodes, 499 edges).

(e) A Random Graph with power law graph (200 nodes, 399 edges).

(f) A Copying Model graph (200 nodes, 199 edges).

(g) A Forest Fire graph (200 nodes, 505 edges).

(h) A Kronecker graph (167 nodes, 264 edges).

Fig. 1. Graphical representation of the network models generated.

3. Network metrics

To assess the suitability of network topologies for the local energy exchange we consider requirements and metrics that the new networks need to satisfy. The metrics considered here come from the set of traditional CNA metrics and from the results obtained in the analysis of medium and low voltage power grids [10]. The novelty resides in defining bounds for the values of such metrics and evaluating the satisfaction of such metrics by the evolution/growth models of networks presented in Section 2. We set two main categories of requirements: qualitative and quantitative desiderata that the network should satisfy.

The main **qualitative requirement** we envision for the future distribution network is based on the modularity of the network topology. In the power system domain, modularity is advocated as a solution that provides benefits by reducing the uncertainties in energy demand forecasting and the costs for energy generation plants, as well as the risks of technological and regulatory obsolescence [3]. Modularity is usually required not only in the energy sector, but more generally in the design and creation of products or organizations [32]. It is also a principle that is promoted in the innovation of complex systems for the benefits it provides in terms of reduced design and development time, adaptation and recombination [33].

Table 2
Metrics classification.

Metric	Efficiency	Resilience	Robustness
CPL	✓		
CC	✓		
Avg. betweenness		✓	
Betw. coeff. of variation		✓	
Rob_N			✓
$APL_{10\text{th}}$	✓	✓	

We assess the modularity of a network as the ability of building the network using a self-similar recurrent approach and having a repetition of a kind of pattern in its structure.

As a global statistical tool, **quantitative requirements** are even more useful as they give a precise indication of network properties. Here are the relevant ones when considering efficiency, resilience and robustness of a power system. For the formal definition of the main CNA properties we refer to well-known literature on the topic e.g., Refs. [6,7].

- *The Characteristic Path Length (CPL)* [26] *is lower or equal to the natural logarithm of the order of the graph:* $CPL \leq ln(N)$, where N is the *order* of the graph. The requirement deals with providing, generally, a limited path when moving from one node to another. In the grid this provides for a network with limited losses in the paths used to transfer energy from one node to another.
- *The Clustering Coefficient (CC)* [26] *is* 5 *times higher than that of a corresponding random graph (RG) with the same order and size*: $CC \geq 5 \times CC_{RG}$. Watts and Strogatz [15] show that small-world networks have clustering coefficients such that $CC \gg CC_{RG}$. Here we require a similar condition, although less strong by using a constant multiplication factor of 5. This requirement is proposed in order to guarantee a local clustering among nodes, since it is more likely that energy exchanges occur at a very local scale (e.g., neighborhood) when small-scale distributed energy resources are broadly implemented.
- *Betweenness-related requirements:*
  - *A low value for average betweenness* [34] *in terms of order of the graph* $\overline{\upsilon} = \frac{\overline{\sigma}}{N}$, where $\overline{\sigma}$ is the average betweenness of the graph and N is the *order* of the graph. For the Internet, Vázquez et al. [35] have found for this metric $\overline{\upsilon} \approx 2.5$. The Internet has proved successful in tolerating failures and attacks [36,37], therefore we require a similar value for this metric for the future grid.
  - *A coefficient of variation for betweenness* $c_\upsilon = \frac{s}{\bar{x}} < 1$ where s is the sample standard deviation and $\bar{x}$ is the sample mean of betweenness. Usually distributions with $c_\upsilon < 1$ are known as low-variance ones.

  The above two requirements are generally considered to provide network resilience by limiting the number of critical nodes that have a high number of minimal paths traversing them. These properties provide distributions of shortest paths which are more uniform among all nodes.
- *An index for robustness such that* $Rob_N \geq 0.45$. Robustness is evaluated with a random removal strategy and a node degree-based removal strategy, by computing the average of the *order* of the maximal connected component (MCC) of the graph between two situations when the 20% of the nodes of the original graph are removed [10]. It can be written as $Rob_N = \frac{|MCC_{Random20\%}|+|MCC_{NodeDegree20\%}|}{2}$. Such a requirement is about twice the value observed for current medium voltage grids and 33% more than the value of the low voltage samples [10].
- *A measure of the cost related to the redundancy of paths available in the network:* $APL_{10\text{th}} \leq 2 \times CPL$.

$$APL_{10\text{th}} = \frac{1}{|G^{\star}|} \sum_{v_i \in G^{\star}, v_j \in G^{\star\prime}} d(v_i, v_j, 10) \tag{1}$$

where

$$d(v_i, v_j, 10) = \begin{cases} l_{v_i, v_j, 10} & \text{if } |P(v_i, v_j)| \geq 10 \\ \max(l_{v_i, v_j}) \in P(v_i, v_j) & \text{if } |P(v_i, v_j)| \leq 10 \end{cases} \tag{2}$$

and $P(v_i, v_j)$ is the set containing the paths between v_i and v_j; $l_{v_i, v_j, 10}$ is the length of the 10th minimal length of the paths between v_i and v_j. With this metric we consider the cost of having redundant paths available between nodes. In particular, we evaluate the 10th shortest path (i.e., the shortest path when the nine best ones are not considered) by covering a random sample of the nodes in the network (40% of the nodes, half of which represent source nodes, $G^{\star}$ in Eq. (1), and the other half represents destination nodes, $G^{\star\prime}$ in Eq. (1)). The values for the paths considered are then averaged. In the case where there are less than ten paths available, the worst-case path between the two nodes is considered. This last condition gives not completely significant values when applied to networks with small connectivity (i.e., in absence of redundant paths).

We categorize the above quantitative metrics into three macro categories with respect to how they affect the power grid, and measure the metrics' goodness from a topological point of view: through the network's efficiency in the transfer of energy, its resilience in providing alternative paths if part of the network is compromised/congested, and its robustness against failures in network connectivity. Table 2 summarizes the property each metric assesses. Each metric gives a specific contribution and all the metrics together cover all the properties a smart grid infrastructure should have.

4. Generating smart grids

The baseline network for comparing possible evolutions must be the real current power grid network. Therefore, we use actual samples from the medium and low voltage network of the Northern Netherlands (for a complete description of the data we refer to Ref. [10]).

Table 3 in Ref. [13] summarizes the values for the network metrics applied on the Dutch network samples. We notice that the average degree of the medium and low voltage samples scores almost constantly around $\langle k \rangle \approx 2$, independently of the *order* of the network. In the low voltage networks we see a tendency towards the increase of the characteristic path length, with a value of about 18 when the *order* and *size* tend towards 200 nodes and edges, respectively. This metric does not have the same clear tendency for the medium voltage samples. Considering the clustering coefficient there is a general rule appears: a null value for the low voltage samples and small, but at least significant, values for the medium voltage samples. These differences in both characteristic path length and clustering coefficient come from the difference in topology of the two networks. A low voltage network is almost a non-mashed network which resembles (for certain samples) trees or closed chains with longer paths on average, especially for networks with bigger *order*. On the other hand, the medium voltage network is more meshed (despite having the same average node degree) with more connections that act as "shortcuts". It also has some redundancy in the connections between the neighborhood of a node, which implies a more significant clustering coefficient compared to the low voltage network. The analysis of the robustness metric shows generally poor scores that decrease while the samples increase in *order*, at least for the low voltage networks, while the tendency is not clear for the medium voltage samples considered. A common behavior for the medium voltage samples is the problem they experience with regard to the maximal component connectivity, when the 20% of the nodes with the highest degree are removed from the network, the *order* of the MCC falls to 4.56%, 3.66% and 3.96% of its initial value, respectively, for the *Small*, *Medium* and *Large* sample. Considering the additional effort required when the first nine shortest paths are not available, we see a general increase especially for the low voltage samples, where the $APL_{10\text{th}}$ increases three times for the *Large* sample analyzed; the increase is still present in medium voltage, but it is limited compared to the low voltage samples. It is again an indication that the medium voltage provides more efficient alternative paths to connect nodes. An exception in the results is the low voltage *Medium* size sample: here the 10th average path length is very close to the traditional characteristic path length. This is due to the absence of alternative paths, therefore the only paths between nodes are at the same time the best and worst case. Such aspect reinforces the idea of a low voltage network with a fixed structure (similar to chains or trees) and a limited redundancy.

Considering the betweenness-related metrics shown in Table 4 in Ref. [13], one notices an increase in the average betweenness as the samples become more numerous in the two segments of the network (i.e., medium voltage and low voltage). This same tendency is present also in the average betweenness/*order* ratio: the biggest samples in terms of *order* both of low voltage and medium voltage score highest. In particular, the *Large* sample belonging to the low voltage is almost twice the value of the biggest sample of the medium voltage. Again this can be justified by the tree-like structure of the low voltage sample, for which nodes responsible for the paths that enable sub-trees or sub-chains to be connected are the highest scoring for betweenness. This highly increases the average betweenness (while the mode is usually null). The coefficient of variation is above one for all the big samples and reaches almost three for the biggest sample belonging to the medium voltage network.

Model parameters

To model the future power grid, we compare network topologies that evolve in *size* (M) and *order* (N). In particular, we consider the increase of the average node degree ($\langle k \rangle = \frac{2M}{N}$). The evolution implies new cables and costs. For the Random Graph, small-world, preferential attachment and R-MAT models, we consider an evolution in the magnitude of the average node degree of $\approx$2 then $\approx$4 and $\approx$6. For models that do not allow explicit settings of both *size* and *order*, we operate on other parameters available that generate comparable networks. Each of the models introduced in Section 2 is defined by a set of specific parameters. For the details of the parameters used in generating each model we refer to Ref. [13].

Model generation

Each network model described is generated and analyzed according to the significant power grid metrics described in Section 3. We begin with the models for which it is possible to explicitly assign *order* and *size* (or one of these quantities and the average node degree); we then proceed analyzing the other models that do not explicitly allow to set the average node degree parameter.

Model generation implementation and metrics computation. The generated topologies are obtained using the Stanford Network Analysis Project (SNAP)[1] library that enables the generation of the network topologies described in Section 2. The analysis of the generated graphs according to the metrics described in Section 3 is performed with ad-hoc created

1 http://snap.stanford.edu/.

Table 3
Metrics for small-world (SW), preferential attachment (PA), Random Graph (RG) and R-MAT models with average node degree ≈2.

Network type	Model	Order	Size	Avg. deg.	CPL	CC	Removal robustness (Rob_N)	Redundancy cost (APL_{10th})
LV-Small	SW	20	20	2.000	4.053	0.00000	0.330	7.580
LV-Medium	SW	90	90	2.000	11.820	0.01593	0.167	12.932
LV-Large	SW	200	201	2.010	17.397	0.01083	0.109	21.544
MV-Small	SW	250	250	2.000	24.237	0.00000	0.087	24.534
MV-Medium	SW	500	501	2.004	28.084	0.00000	0.057	35.413
MV-Large	SW	1000	1001	2.002	47.077	0.00000	0.040	60.074
LV-Small	PA	20	19	1.900	2.579	0.00000	0.349	2.800
LV-Medium	PA	90	89	1.978	4.315	0.00000	0.263	4.471
LV-Large	PA	200	199	1.990	6.523	0.00000	0.206	6.375
MV-Small	PA	250	249	1.992	5.426	0.00000	0.245	5.570
MV-Medium	PA	500	499	1.996	5.705	0.00000	0.231	5.745
MV-Large	PA	1000	999	1.998	6.976	0.00000	0.187	6.908
LV-Small	RG	17	21	2.471	2.938	0.07451	0.390	7.472
LV-Medium	RG	78	92	2.359	5.987	0.03547	0.418	10.974
LV-Large	RG	172	207	2.407	6.254	0.00736	0.354	10.796
MV-Small	RG	224	259	2.313	7.269	0.00000	0.322	12.002
MV-Medium	RG	435	516	2.372	8.380	0.00138	0.321	12.818
MV-Large	RG	863	1026	2.378	9.061	0.00070	0.328	13.446
LV-Small	R-MAT	27	31	2.296	3.615	0.00000	0.356	7.830
LV-Medium	R-MAT	88	125	2.841	4.115	0.05688	0.369	6.418
LV-Large	R-MAT	199	261	2.623	5.495	0.00737	0.364	8.774
MV-Small	R-MAT	195	263	2.697	5.629	0.00865	0.378	8.642
MV-Medium	R-MAT	365	523	2.866	5.470	0.01360	0.396	7.646
MV-Large	R-MAT	728	1056	2.901	5.726	0.00589	0.363	7.887

Table 4
Betweenness metrics for small-world (SW), preferential attachment (PA), Random Graph (RG) and R-MAT models with average node degree ≈2.

Network type	Model	Order	Size	Avg. betweenness	Avg. betw/order	Coeff. variation
LV-Small	SW	20	20	62.300	3.115	0.804
LV-Medium	SW	90	90	985.956	10.955	1.307
LV-Large	SW	200	201	3429.720	17.149	1.260
MV-Small	SW	250	250	5881.296	23.525	1.598
MV-Medium	SW	500	501	13980.228	27.960	1.745
MV-Large	SW	1000	1001	47919.616	47.920	2.279
LV-Small	PA	20	19	31.400	1.570	2.344
LV-Medium	PA	90	89	293.400	3.260	3.068
LV-Large	PA	200	199	1089.260	5.446	3.288
MV-Small	PA	250	249	1096.144	4.385	3.972
MV-Medium	PA	500	499	2401.680	4.803	5.049
MV-Large	PA	1000	999	6061.288	6.061	6.240
LV-Small	RG	17	21	31.059	1.827	1.157
LV-Medium	RG	78	92	408.308	5.235	1.126
LV-Large	RG	172	207	938.512	5.456	1.276
MV-Small	RG	224	259	1474.143	6.581	1.265
MV-Medium	RG	435	516	3415.890	7.853	1.204
MV-Large	RG	863	1026	7081.119	8.205	1.264
LV-Small	R-MAT	27	31	70.593	2.615	1.320
LV-Medium	R-MAT	88	125	282.500	3.210	1.540
LV-Large	R-MAT	199	261	937.578	4.711	1.297
MV-Small	R-MAT	195	263	959.118	4.919	1.395
MV-Medium	R-MAT	365	523	1692.910	4.638	1.581
MV-Large	R-MAT	728	1056	3633.473	4.991	2.004

software based on the JAVA graph library JGraphT.[2] The versions of SNAP and JGraphT software libraries used are respectively v10.10.01 and v0.8.1.

Comparison of models with average node degree $\langle k\rangle \approx 2$

The results for the metrics with average degree $\langle k\rangle \approx 2$ for the small-world, preferential attachment, Random Graph and R-MAT models score quite poorly, cf. Table 3. Low values for the metrics are due to the small connectivity the networks show. Especially, we highlight the low results of the small-world model under these conditions.

The betweenness analysis, whose results are presented in Table 4, shows an average for each node that increases with the size of the graph.

2 http://www.jgrapht.org/.

Table 5
Metrics for small-world (SW), preferential attachment (PA), Random Graph (RG) and R-MAT models with average node degree ≈4.

Network type	Model	Order	Size	Avg. deg.	CPL	CC	Removal robustness (Rob_N)	Redundancy cost (APL_{10th})
LV-Small	SW	20	39	3.900	2.289	0.26000	0.721	4.720
LV-Medium	SW	90	177	3.933	3.652	0.14646	0.780	6.032
LV-Large	SW	200	399	3.990	4.407	0.15367	0.767	6.631
MV-Small	SW	250	498	3.984	4.566	0.12581	0.779	6.836
MV-Medium	SW	500	1000	4.000	5.067	0.10681	0.764	7.231
MV-Large	SW	1000	1998	3.996	5.749	0.10879	0.781	7.910
LV-Small	PA	20	37	3.700	2.263	0.47341	0.554	4.380
LV-Medium	PA	90	177	3.933	2.910	0.11216	0.426	4.788
LV-Large	PA	200	397	3.970	3.322	0.09566	0.448	5.047
MV-Small	PA	250	497	3.976	3.504	0.08400	0.419	4.998
MV-Medium	PA	500	997	3.988	3.687	0.03929	0.401	5.232
MV-Large	PA	1000	1997	3.994	4.211	0.01536	0.401	5.678
LV-Small	RG	20	40	4.000	2.079	0.17667	0.733	4.350
LV-Medium	RG	87	180	4.138	3.174	0.03418	0.735	5.368
LV-Large	RG	199	400	4.020	3.869	0.03064	0.734	6.107
MV-Small	RG	247	500	4.049	4.057	0.01681	0.740	6.432
MV-Medium	RG	494	1000	4.049	4.495	0.00823	0.749	6.670
MV-Large	RG	987	2001	4.055	5.062	0.00359	0.738	7.150
LV-Small	R-MAT	30	59	3.933	2.517	0.27360	0.579	4.511
LV-Medium	R-MAT	105	250	4.762	3.019	0.13039	0.581	4.490
LV-Large	R-MAT	227	504	4.441	3.619	0.04683	0.601	5.302
MV-Small	R-MAT	230	496	4.313	3.736	0.02940	0.626	5.381
MV-Medium	R-MAT	420	1004	4.781	3.915	0.00450	0.591	5.249
MV-Large	R-MAT	932	2039	4.376	4.562	0.00875	0.690	6.251

Table 6
Betweenness metrics for small-world (SW), preferential attachment (PA), Random Graph (RG) and R-MAT models with average node degree ≈4.

Network type	Model	Order	Size	Avg. betweenness	Avg. betw/order	Coeff. variation
LV-Small	SW	20	39	24.900	1.245	0.654
LV-Medium	SW	90	177	235.244	2.614	0.653
LV-Large	SW	200	399	683.780	3.419	0.703
MV-Small	SW	250	498	897.568	3.590	0.653
MV-Medium	SW	500	1000	2043.600	4.087	0.706
MV-Large	SW	1000	1998	4762.808	4.763	0.677
LV-Small	PA	20	37	23.100	1.155	1.505
LV-Medium	PA	90	177	170.644	1.896	2.219
LV-Large	PA	200	397	463.060	2.315	2.733
MV-Small	PA	250	497	611.520	2.446	3.017
MV-Medium	PA	500	997	1342.864	2.686	3.484
MV-Large	PA	1000	1997	3179.750	3.180	3.450
LV-Small	RG	20	40	23.600	1.180	0.807
LV-Medium	RG	87	180	196.345	2.257	0.850
LV-Large	RG	199	400	589.849	2.964	0.889
MV-Small	RG	247	500	766.389	3.103	0.857
MV-Medium	RG	494	1000	1768.757	3.580	0.972
MV-Large	RG	987	2001	4068.393	4.122	0.942
LV-Small	R-MAT	30	59	44.000	1.467	1.342
LV-Medium	R-MAT	105	250	223.733	2.131	1.695
LV-Large	R-MAT	227	504	609.419	2.685	1.493
MV-Small	R-MAT	230	496	650.374	2.828	1.468
MV-Medium	R-MAT	420	1004	1285.786	3.061	1.652
MV-Large	R-MAT	932	2039	3422.348	3.672	1.506

Comparison of models with average node degree $\langle k \rangle \approx 4$

Table 5 shows the results for small-world, preferential attachment, Random Graph and R-MAT models with an average degree $\langle k \rangle \approx 4$. One notices that the scores for the metrics improve compared to the $\langle k \rangle \approx 2$ case. The average over the characteristic path length of all the samples reduces from around 10 to a value that is slightly less than 5. The clustering coefficient has values that are significant and all positive. The small-world model scores best in this specific metric, since it relies on the lattice topology that, with an average degree of 4, connects each node with four neighbors. In particular, 3-triangle structures emerge in each neighborhood of a node (of course before the rewiring process takes place). This provides a substantial contribution to the quite high clustering coefficient. A graphical comparison for the *Large* sample for medium voltage considering the characteristic path length, clustering coefficient and robustness metrics are shown in Fig. 2.

When analyzing network betweenness, we see a general improvement in the metrics compared to the $\langle k \rangle \approx 2$ case, cf. Table 6. The most important improvement is for the small-world model which, with approximately 4 connections per node, substantially reduces the average betweenness by a factor of 10 compared to the $\langle k \rangle \approx 2$ case. A graphical comparison for

Table 7
Metrics for small-world (SW), preferential attachment (PA), Random Graph (RG) and R-MAT models with average node degree ≈6.

Network type	Model	Order	Size	Avg. deg.	CPL	CC	Removal robustness (Rob_N)	Redundancy cost ($APL_{10\text{th}}$)
LV-Small	SW	20	59	5.900	1.816	0.33250	0.775	3.470
LV-Medium	SW	90	266	5.911	2.809	0.20131	0.794	4.508
LV-Large	SW	200	598	5.980	3.324	0.13596	0.797	4.895
MV-Small	SW	250	747	5.976	3.486	0.14477	0.798	5.039
MV-Medium	SW	500	1494	5.976	3.968	0.14477	0.799	5.518
MV-Large	SW	1000	2996	5.992	4.429	0.14854	0.797	5.905
LV-Small	PA	20	54	5.400	1.868	0.34839	0.749	3.460
LV-Medium	PA	90	264	5.867	2.466	0.16601	0.742	3.933
LV-Large	PA	200	594	5.940	2.854	0.08772	0.671	4.130
MV-Small	PA	250	744	5.952	2.926	0.08676	0.705	4.257
MV-Medium	PA	500	1495	5.980	3.185	0.05017	0.667	4.481
MV-Large	PA	1000	2994	5.988	3.487	0.03335	0.679	4.664
LV-Small	RG	20	60	6.000	1.684	0.29599	0.775	3.370
LV-Medium	RG	90	270	6.000	2.640	0.06987	0.791	4.298
LV-Large	RG	200	600	6.000	3.141	0.03991	0.777	4.693
MV-Small	RG	249	750	6.024	3.230	0.01934	0.793	4.884
MV-Medium	RG	499	1500	6.012	3.620	0.00976	0.792	5.284
MV-Large	RG	998	3000	6.012	4.022	0.00544	0.791	5.662
LV-Small	R-MAT	32	87	5.438	2.194	0.21179	0.760	3.945
LV-Medium	R-MAT	123	374	6.081	2.926	0.08173	0.717	4.377
LV-Large	R-MAT	249	759	6.096	3.165	0.04444	0.736	4.622
MV-Small	R-MAT	236	747	6.331	3.143	0.04982	0.746	4.389
MV-Medium	R-MAT	466	1512	6.489	3.427	0.04365	0.743	4.805
MV-Large	R-MAT	925	3035	6.562	3.742	0.02560	0.723	4.925

the results of the *Large* sample for medium voltage type considering the average betweenness/*order* ratio and the coefficient of variation metrics is shown in Fig. 3.

Comparison of models with average node degree $\langle k \rangle \approx 6$

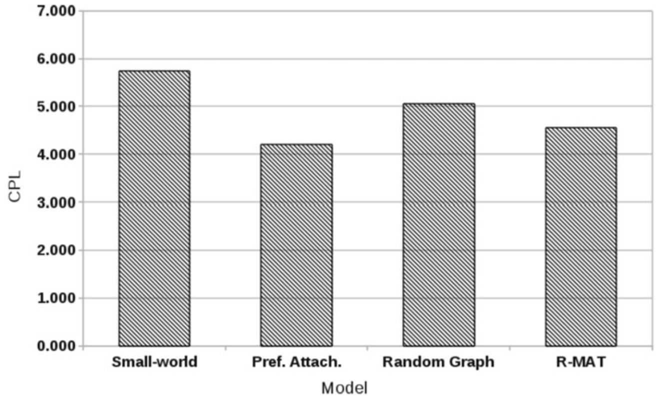
Table 7 shows the results for small-world, preferential attachment, Random Graph and R-MAT models with an average degree $\langle k \rangle \approx 6$. The scores for the metrics considered improve even more with respect to those of Tables 3 and 5. The characteristic path length of all the samples has reduced to a value that, considering the average over all the samples with $\langle k \rangle \approx 6$, is about 3; yet this is 2 hops lower than the situation with $\langle k \rangle \approx 4$. The same tendency for the clustering coefficient found for samples in Table 5 applies to this situation, too. The small-world model scores highest since the neighbors of a node have nine connections with each other (before rewiring), thus contributing to a high coefficient.

Having increased the average degree to 6 brings benefits to the betweenness statistics too, cf. Table 8. The benefits on the average betweenness/*order* ratio are about 25% higher than in the $\langle k \rangle \approx 4$ situation; this ratio therefore is now very close to the experimental values that have been found for the Internet (i.e., ≈2.5).

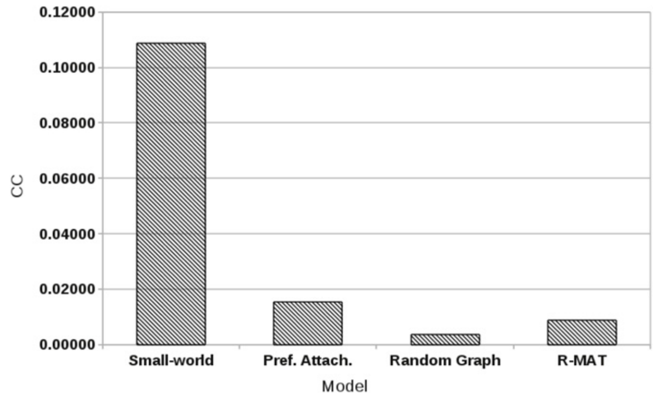
A more thorough analysis of these results and a detailed comparison and analysis of the models independent from average node degree have been performed and the comprehensive results are available in Ref. [13].

Comparing the generated topologies with the physical ones

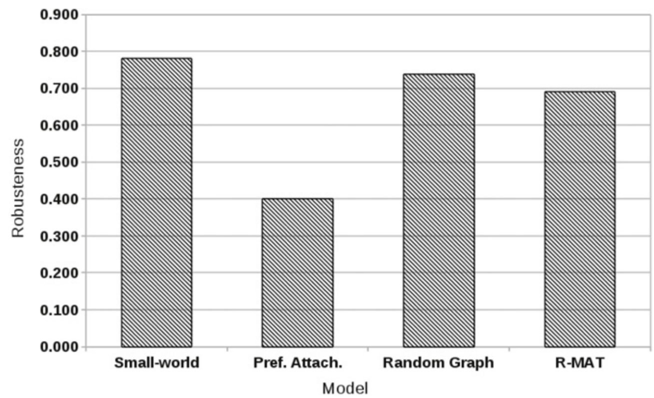
The analysis of the Northern Netherlands grid shows an average degree almost constant of about $\langle k \rangle \approx 2$. In terms of average node degree, the situation is similar for the high voltage grid based on the data describing the Eastern and Western high voltage U.S. power grid and the U.S. Western high voltage power grid. Therefore we consider it to be fair to compare the generated models with similar average degree, the Copying Model ones and the Random Graphs with power-law in node degree distribution with average node degree $\langle k \rangle \approx 2$. Generated models, except the model based on Random Graph with power-law, score better than the physical topologies for all the metrics considered; the characteristic path length scores half for the R-MAT and Copying Model cases in comparison to the real data. Also synthetic networks are more robust than the real data samples: R-MAT and Random Graph score constantly above 0.3 for robustness metric, while real data hardly obtain this value. Clustering coefficients are quite similar since in this configuration with limited connectivity having triangle structures in the network is rare, however we see that the R-MAT model has almost always significant clustering coefficient values. An exception is the small-world model which scores almost always worse than the real data samples, in fact, under this situation of such average node degree it is actually not fully correct to consider this synthetic topology a "small-world". The same sort of considerations can be done considering betweenness values: except the small-world model, all the other synthetic ones score better for the average betweenness/*order* ratio metric, while for the coefficient of variation the situation is similar. If one considers the satisfaction of the desiderata for the actual samples of the Dutch Medium and low voltage grid, summarized in Table 9, we notice that all parameters are not satisfied. However, networks generated according to the models with almost the same average node degree (networks with $\langle k \rangle \approx 2$ in Table 20 in Ref. [13], and networks based on Random Graph with power-law based on data from Eastern and Western high voltage U.S. power grid and the U.S. Western



(a) Characteristic path length.



(b) Clustering coefficient.

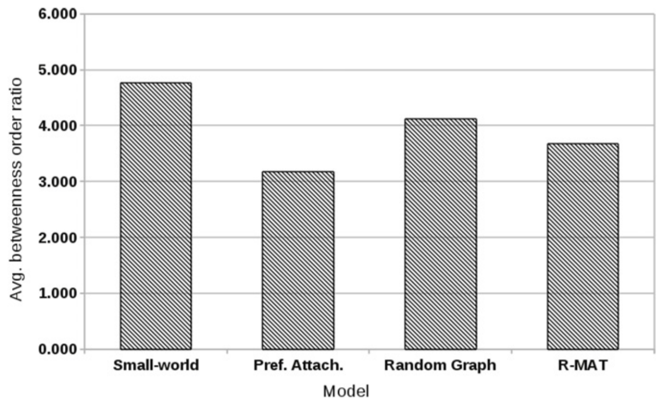


(c) Removal robustness.

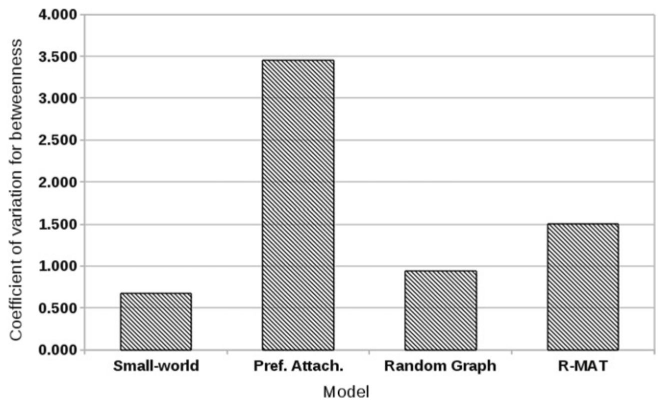
Fig. 2. Results for metrics for the *Large* sample of medium voltage network type with average node degree ≈4.

high voltage power grid in Table 22 in Ref. [13]) do not satisfy all the desiderata as well. Therefore, this highlights that the first ingredient for the next generation of grids to enable local energy exchange is an increased connectivity.

Increasing the average node degree naturally provides for better values for the network metrics, as shown in Table 20 in Ref. [13]. The case of the small-world model is emblematic. The $\langle k \rangle \approx 2$ case scores extremely poorly as there are not enough "shortcuts" in the network so that they cannot improve much the characteristic path length. Actually, under such small average degree, the condition Watts and Strogatz impose for their model is not completely satisfied (i.e., $n \gg k \gg ln(n) \gg 1$, where k is the average node degree and n is the *order* of the graph). When we move closer to satisfying the small-world condition by increasing the average node degree, the value of the metrics suddenly change and the models score extremely high. The small-world scores best for the clustering property and resilience to failures in



(a) Betweenness to *order* ratio.



(b) Betweenness coefficient of variation.

Fig. 3. Results for metrics for the *Large* sample of medium voltage network type with average node degree ≈4.

Table 8
Betweenness-related metrics for small-world (SW), preferential attachment (PA), Random Graph (RG) and R-MAT models with average node degree ≈6.

Network type	Model	Order	Size	Avg. betweenness	Avg. betw/order	Coeff. variation
LV-Small	SW	20	39	15.800	0.790	0.581
LV-Medium	SW	90	177	163.778	1.820	0.555
LV-Large	SW	200	399	464.330	2.322	0.617
MV-Small	SW	250	498	621.488	2.486	0.609
MV-Medium	SW	500	1000	1479.404	2.959	0.565
MV-Large	SW	1000	1998	3441.742	3.442	0.564
LV-Small	PA	20	37	15.900	0.795	1.292
LV-Medium	PA	90	177	133.378	1.482	2.640
LV-Large	PA	200	397	374.970	1.875	2.401
MV-Small	PA	250	497	485.352	1.941	2.514
MV-Medium	PA	500	997	1095.116	2.190	2.894
MV-Large	PA	1000	1997	2447.594	2.448	3.283
LV-Small	RG	20	40	14.700	0.735	0.662
LV-Medium	RG	87	180	151.489	1.683	0.809
LV-Large	RG	199	400	431.090	2.155	0.835
MV-Small	RG	247	500	563.839	2.264	0.710
MV-Medium	RG	494	1000	1328.405	2.662	0.745
MV-Large	RG	987	2001	3051.922	3.058	0.771
LV-Small	R-MAT	30	59	38.000	1.188	0.989
LV-Medium	R-MAT	105	250	247.008	2.008	1.351
LV-Large	R-MAT	227	504	550.538	2.211	1.352
MV-Small	R-MAT	230	496	530.093	2.246	1.357
MV-Medium	R-MAT	420	1004	1169.382	2.509	1.506
MV-Large	R-MAT	932	2039	2599.496	2.810	1.731

Table 9
Desiderata parameter compliance of real samples of the Northern Netherlands grid.

Desiderata	Northern Netherlands medium and low voltage samples
Modularity	✗
$CPL \leq ln(N)$	✗
$CC \geq 5 \times CC_{RG}$	✗
$\overline{\upsilon} = \frac{\overline{\sigma}}{N} \approx 2.5$	✗
$c_v \leq 1$	✗
$Rob_N \geq 0.45$	✗
$APL_{10\text{th}} \leq 2 \times CPL$	≈

Table 10
Comparison of generated topologies for varying average node degree.

Avg. node degree transition	Average metric improvement (%)		
	CPL	CC	Robustness
$\langle k \rangle \approx 2 \rightarrow \langle k \rangle \approx 4$	61.7	941.6	128.5
$\langle k \rangle \approx 4 \rightarrow \langle k \rangle \approx 6$	18.0	11.8	19.6

Table 11
Satisfaction of modularity, performance and cabling cost for generated models.

Network model	Avg. node deg. $\langle k \rangle \approx 2$	Avg. node deg. $\langle k \rangle \approx 4$	Avg. node deg. $\langle k \rangle \approx 6$
Small-world	✓✓	✓✓✓	✓✓
Preferential attachment	✓	✓✓	✓
Random Graph	✓	✓✓	✓
R-MAT	✓✓	✓✓	✓✓

$\langle k \rangle \approx 4$ situations. Under these conditions also the betweenness values are quite concentrated around the mean with a coefficient of variation not exceeding the unit.

Comparing the average values of the generated models for increasing node degree, one notices a natural improvement of the metrics, cf. Table 10. In fact, we have a reduction in characteristic path length of about 60% and an increase in the clustering coefficient of one order of magnitude; at the same time the robustness doubles. With $\langle k \rangle \approx 6$ the improvement in the metrics is less prominent, i.e., between 10% and 20%. From the comparison of the metric results in Table 20 in Ref. [13], one sees that the small-world model almost always satisfies the desired requirement from a quantitative point of view when the average node degree is at least 4. From a qualitative point of view, the small-world model shows some modularity being generated starting from a regular lattice and then rewiring a certain fraction of the edges.

The models which are independent from average node degree perform generally worse than the other models. The adherence to the target values is shown in Table 22 in Ref. [13]. There is a general prevalence of requirement dissatisfaction, especially for parameters involving betweenness.

From the topological analysis one can see that between the models analyzed when there is a minimal connectivity ($\langle k \rangle \approx 4$ or $\langle k \rangle \approx 6$) the small-world stands out, cf. Table 20 in Ref. [13]. In Table 11, the models with explicit dependence on node degree are once again compared by assigning a "tick" sign (✓) for the fulfillment of each of the following properties: qualitative topological parameters (i.e., modularity), quantitative topological parameters (Table 20 in Ref. [13]) and the thrift in network realization (e.g., addition of cables which represent a cost). The latter is just an estimation; a more detailed analysis of the cost in realizing a network of to medium or low voltage with a certain size (i.e., *Small*, *Medium* or *Large*) and the economic benefits in electricity distribution arising from the enhanced connectivity is provided in Section 5. From Table 11, we conclude that networks generated with small-world model with average degree $\langle k \rangle \approx 4$ provide the best balance between modularity, performance and thrift for the future power grid.

5. Economic considerations

Traditionally, the problem of evaluating the expansion of an electrical system is a complex task that involves both the use of modeling (usually based on operation research optimization techniques and linear programming [38]) and the experience and vision of experts in the field supported in their decisions by computers. With more distributed generating facilities at local scale, traditional methods have limits and need to be modified or updated to take into account the new scenarios the smart grid framework brings into play. The models that we have so far analyzed as being candidates for the vision of the

Table 12
Cabling cost for $\langle k \rangle \approx 2$ synthetic samples for Low voltagenetworks.

Sample type	Size	Cost (thousand euro)
Low voltage-Small	≈20	≈30
Low voltage-Medium	≈90	≈78
Low voltage-Large	≈200	≈449

Table 13
Cabling cost for $\langle k \rangle \approx 2$ synthetic samples for medium voltage networks.

Sample type	Size	Cost (millions euro)
Low voltage-Small	≈250	≈32
Low voltage-Medium	≈500	≈42
Low voltage-Large	≈1000	≈43

future smart grid need also to be evaluated from the economic point of view. How much will it cost to generate electrical infrastructures according to these models? What is the actual cost of adding a physical edge to the topology?

The cost of adding edges

One important difference that a physical infrastructure such as the power grid has compared to the WWW or social networks is the physical presence of cables that have to connect the medium voltage substations or low voltage end-users' generating units. If establishing a link from a Web page to another one is free, each increase in connectivity in the power grid implies costs in order to build or upgrade the substation or end-user premise involved and the cables required for the connection. To assess these costs in the medium and low voltage infrastructure, we consider a simple relation where the cost of cabling and cost of substations are added:

$$C_{imp} = \sum_{j=1}^{N} Ssc_j + \sum_{i=1}^{M} Cc_i \tag{3}$$

where C_{impl} stands for cost for implementation, Ssc_j is the adaptation cost for the substation j and Cc_i is the cost for the cable i. The cost of the cable can be expressed as a linear function of the distance the cable i covers: $Cc_i = C_{uc} \cdot l$ where C_{uc} is the cable cost per unit of length and l is the length of the cable. Several types of cables exist which are used for power transmission and distribution with varying physical characteristics and costs. In addition, the cost for installation can also vary significantly given the difficulty to perform underground or aerial line installations due to the geographical characteristics of the territory [39]. In the present work, to provide an initial estimate, we simply consider cabling costs and ignore substation ones. While the pricing for the former is directly tied to the topology and length of the links, the pricing for the latter is too dependent on other factors such as the cost of land to expand a substation. This cost varies considerably given urban or rural locations and between different regions. As a source of data for cable type and pricing, we have been provided (courtesy of Enexis B.V. The Netherlands) with cable characteristics and prices, together with topological information, for 11 network samples belonging to the low voltage network and 13 samples belonging to the medium voltage of the Northern Netherlands, the same from which we extracted the topological properties.

Economic benefits of highly connected topologies

Once the information about cable prices is available, it is possible to estimate the cost for realizing a network with a certain connectivity and whether such networks are able to lower the (economic) barrier towards decentralized energy trading. The results for low voltage networks of *Small*, *Medium* and *Large* types with an average node degree $\langle k \rangle \approx 2$ are shown in Table 12. The results for $\langle k \rangle \approx 4$ and $\langle k \rangle \approx 6$ are about two and three times more expensive, since there is an increase in the number of edges by the same quantity.

For medium voltage, the results for the networks with an average node degree $\langle k \rangle \approx 2$ are shown in Table 13. The results for $\langle k \rangle \approx 4$ and $\langle k \rangle \approx 6$ are just two and three times more expensive since there is an increase in the number of edges by these same factors. The small difference in costs between the *Medium* and *Large* types of networks for medium voltage is related mainly to the different technologies (i.e., cable types) in the cables that are used for the types of networks that we have found in the sample data provided.

Price alone is not enough to describe future scenarios. It is important to investigate how an enhanced connectivity is beneficial to the electricity distribution costs. We have shown the benefits for more connected networks in Section 4; however, those results consider only the topology without any parameter related to the properties of the cables (e.g., resistance and supported current). In order to consider the effects of topology in electricity distribution costs, we have developed a set of metrics [10] (the α and β metrics) that associate topological properties of power grid networks to costs in electricity distribution. We have applied these metrics in the analysis of the medium and low voltage grid of the Northern Netherlands in

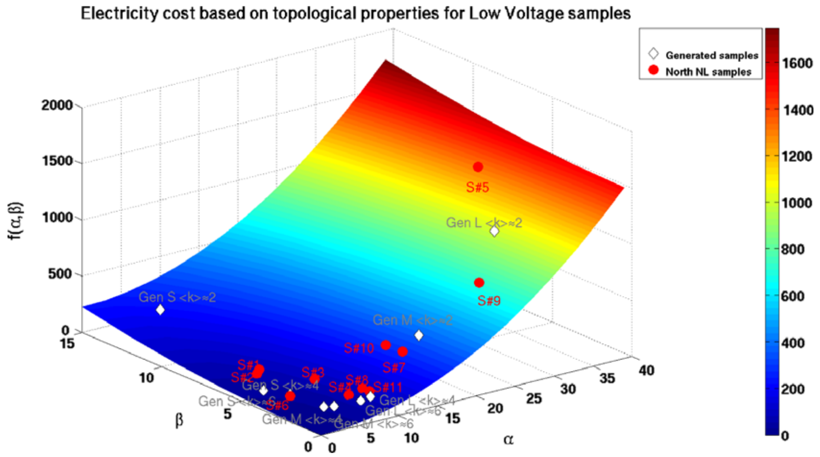


Fig. 4. Comparison of the transport cost between synthetic small-world networks (white diamonds) and Northern Netherlands low voltage samples (red dots).

Ref. [10]. In order to apply these metrics to power grid networks, *weights* are essential, representing physical quantities such as the resistance of the cable and the maximal operating current supported by the cable. Once we have the statistical information about the types and the length of the cables used in a specific type of physical network (i.e., medium or low voltage and its *Small*, *Medium* or *Large* size) it is possible to assign *weights* to the edges of the generated graphs. This is done under the assumption that the same type of cables are used and that the distances covered in general (i.e., statistically) remain the same.

We consider the α and β metrics for networks generated following the small-world model, since it has proven to be the best one in the pure topological analysis (Section 4). For low voltage network, we compute the metrics for networks with an increasing average node degree ($\langle k\rangle \approx 2$, $\langle k\rangle \approx 4$ and $\langle k\rangle \approx 6$). It is not surprising to see the samples with $\langle k\rangle \approx 2$ score poorer than the other networks. The network with *Medium* size scores best and the difference between the network with $\langle k\rangle \approx 6$ and the network with $\langle k\rangle \approx 4$ is limited. Robustness (i.e., the β parameter) for the *Medium* and *Large* size networks reaches a high value just with a sufficient connectivity (i.e., $\langle k\rangle \approx 4$) and more connectivity (i.e., $\langle k\rangle \approx 6$) does not improve much this metric. The samples with *Small* size score better in the α metric, and this is quite reasonable since the paths are limited, of course due to the reduced *order* of the network.

Considering the α and β metrics for the networks generated for the medium voltage, the same tendency appears: once the network is sufficiently connected (i.e., $\langle k\rangle \approx 4$) the metrics score definitely better than the $\langle k\rangle \approx 2$ situation.

Let us compare the α and β metrics of the synthetic networks with the values of the real power grid samples of the Northern Netherlands. Considering the low voltage samples and the synthetic networks designed for this purpose, we generally see an improvement in the metrics, especially in the α values for the $\langle k\rangle \approx 4$ and $\langle k\rangle \approx 6$ networks. In fact, if we do not consider the synthetic networks with $\langle k\rangle \approx 2$; because of the problems of small-world topology with such small connectivity, there is an improvement on average in the α metric for synthetic samples with $\langle k\rangle \approx 4$ of more than 50% compared to the Northern Netherlands samples. In fact, for the α metric from an average of about 13 for the physical samples, the $\langle k\rangle \approx 4$ synthetic ones score about 6. The improvement is more than 60% when considering the $\langle k\rangle \approx 6$ ones where the average for these synthetic networks scores just below 5. There are improvements also in the β metric, although limited. From an average around 4 for the physical samples, the $\langle k\rangle \approx 4$ on average score just below 2.75; while a better result is obtained by $\langle k\rangle \approx 6$ which on average score 2.30 (about 40% improvement). The graphical comparison between Dutch real samples (red dots) and generated samples (white diamonds) is shown in Fig. 4, in which each dot represents a sample in the α, β quadratic function envelope that is chosen as the type of dependence between the topological parameters and electricity transport prices.

Taking into account the Dutch medium voltage samples and the small-world synthetic networks, we see an important improvement in the metrics both in the α and β values for the $\langle k\rangle \approx 4$ and $\langle k\rangle \approx 6$ networks. As already mentioned, synthetic networks with $\langle k\rangle \approx 2$ should not be considered. The improvement on average in the α metric is more than 65% compared to the $\langle k\rangle \approx 4$ synthetic samples (from an average of the α metric about 33 for the physical samples, the $\langle k\rangle \approx 4$ synthetic ones score about 11), and an improvement of more than 75% when comparing to the $\langle k\rangle \approx 6$ ones (the average of the α metric for $\langle k\rangle \approx 6$ synthetic networks scores around 7.3). There are improvements also in the β metric. In particular, from an average around 3.55 for the physical samples the $\langle k\rangle \approx 4$ score on average just below 1.15; a similar result is obtained by $\langle k\rangle \approx 6$, which on average score about 1.2 (more than 65% improvement). The graphical comparison is shown in Fig. 5.

6. Discussion

Watts and Strogatz's small-world model, as shown in Tables 20, 22 in Ref. [13] and Table 11, is the model that captures best the requirements for the new grid compared to the other analyzed, be these dependent on the average node degree

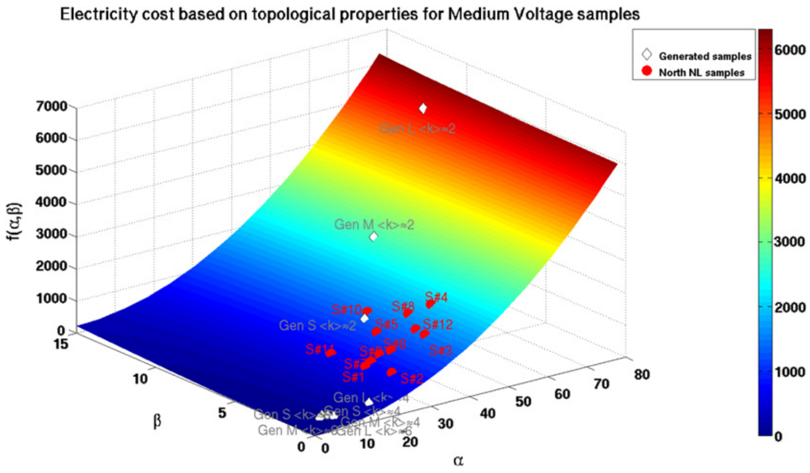


Fig. 5. Comparison for transport cost between synthetic small-world networks (white diamonds) and Northern Netherlands medium voltage samples (red dots).

(preferential attachment, R-MAT and Random Graph) or not (Copying Model, Forest Fire, Kronecker and Power Laws). The tight clustering that these models exhibit provides efficient local distribution with paths that are locally short; at the same time, the shortcuts between the local clusters are the elements that keep the average path extremely limited. These two aspects influence the α parameter which then stays relatively small. At the same time, the small-world model benefits from a general robustness against failures: the absence of big hubs that keep the network together (which are present on the other hand in the power-law-based topologies, for instance) improves the reliability against attacks which help obtaining good scores for the β parameter. More quantitatively, one sees the general improvement in the metrics characterizing both the parameters influencing the losses (i.e., the α parameter) and the reliability of the grid (i.e., the β parameter) while the network becomes more dense, i.e., more edges are added. On average, we see an improvement of at least 50% when comparing the physical samples of Northern Netherlands with the small-world networks with an average degree $\langle k\rangle \approx 4$, while better results are obtained with more density (i.e., $\langle k\rangle \approx 6$) where the improvement are 60% compared to the physical samples. This is indeed beneficial to the power grid and, according to the relationship with the topology, it should translate into a reduction in the costs for electricity distribution since the α and β metrics are composed of essential ingredients influencing electricity distribution price [10].

These benefits come literally at a cost. The network needs more connectivity, therefore costs for extra cabling need to be considered in addition to the cost for upgrading the substations and end-users electricity gateways. A return-on-investment analysis on this aspect is beyond the scope of the present study. Nevertheless, it is interesting to see how, using the α and β metrics, it is possible to consider how a certain physical sample belonging to a certain size category (*Small*, *Medium* and *Large*) would improve in its performance if its topology was arranged according to the principles of a synthetic model and more connections were added accordingly.

The benefits reached for the α and β metrics should translate into a reduction in the cost for electricity transport and distribution, since the parameters that influence these metrics are directly connected to aspects related to costs. However, the significant investment required to add more connectivity in the network might not immediately enable cheaper electricity costs, but, on the contrary, make it more expensive.

On the one hand, dense networks are less vulnerable compared to less dense ones, as we have shown in the quantities that measure robustness to node failures. From a topological perspective, a more robust network reduces the costs (e.g., economic output loss) of potential outages [40]. On the other hand additional costs in equipment have to be taken into account considering, e.g., the electric line protection system and protection coordination of the distribution grid. In our vision, grids need to be more dense than the current ones, therefore more lines need to monitored and protected against currents beyond the specification limits. Methods and principles today mainly applied to the high voltage system (e.g., Ref. [41]) will be inherited by the distribution grid to protect its lines when the distribution grid becomes more complex (e.g., distributed generation) than its current state. Thus, a balance between the additional costs of the protection systems and a reduction in the costs of local blackouts will need to be evaluated when smart grids will be implemented on the field.

7. Related work

Network evolution, growth and shrink processes have been studied in the field of statistical physics to characterize and model how network phenomena in nature and in man-made infrastructures emerge [42,43], in social sciences to evaluate the social network evolution [44,45], and also by computer scientists to study the evolution of the Internet [46,35]. However, all these works only focus on the analysis and modeling of the existing grids rather than proposing how a new network topology should be (or should evolve) to maximize the benefits for members of such a network.

Complex Network Analysis related work in the power grid domain takes into account the high voltage level usually to analyze the structure of the network without considering in detail the physical properties of the power lines. In Ref. [11], we have analyzed several works that investigate power grid properties using CNA approach. There are basically two main categories of works with the following objectives: (1) understand the intrinsic property of grid topologies and compare them to other types of networks, assessing the existence of properties such as small-world or scale-free e.g., Refs. [26,15,16]; and (2) better understand the behavior of the network when failures occur (i.e., edge or node removal) and analyze the topological causes of black-out spread and cascading failures of power lines, e.g., Refs. [47,23,48,49]. Few studies in the CNA landscape consider the possibility of using the insight gained through the analysis to help the power grid design. These few cases consider the addition of lines in the network to assess the increase in the reliability of the entire power grid. Examples are the study of the Italian high voltage grid [50] and the study of improvement by line addition in Italian, French and Spanish grids [51]. Also Holmgren [52] uses the CNA to understand which grid improvement strategies are most beneficial showing improvement of typical CNA metrics (e.g., path length, average degree, clustering coefficient, network connectivity), although for a very small graph (fewer than 10 nodes) when different edges and nodes are added to the network. Broader is the work of Mei et al. [53], where a self-evolution process of the high voltage grid is studied with CNA methodologies. Wang et al. [54,55] study the power grid to understand the kind of communication system and the related network topologies needed to support the decentralized control required by the new power grid applying CNA techniques. The simulation results are compared to the real samples of high voltage U.S. power grid and synthetic reference models from the IEEE literature. The effects of distributed and erratic generation from renewable sources, which is a key feature in the future grid is considered in Ref. [56] where cascades in several IEEE bus networks are considered due to the overload of power lines analyzed with a DC flow power model. In Ref. [57] the topic of renewable-based generation and their location on the grid is analyzed from a topological (i.e., page rank-based location of the nodes to attach the generators) and from an electrical point of view (i.e., effects on the stability of the grid). The authors show, for the Polish high voltage grid that a small amount the renewable generation is beneficial, but once a certain threshold is surpassed, it is a threat for the stability of the grid. CNA is not generally used as a design tool to propose new topologies for the future smart grid, especially at medium and low voltage level, as we use in this paper, where we also assess the benefits in terms of economical improvement.

8. Conclusions

In an evolving electricity sector with end-users able to produce their own energy and sell it on a local-scale market, the grid plays the essential enabling role of supporting infrastructure. Local scale energy exchange is in fact beneficial for several aspects, such as the increase in renewable-based energy production, the possibility for the end-user to make a profit by selling surplus energy and, not less important, making a step forward towards the unbundling of the electricity sector. We studied how different topologies inspired from technological and social network studies have different properties and may (or not) be adequate for the future smart grid networks. We showed that among the various models analyzed, the small-world model appears to have many appropriate characteristics, according to a set of topological metrics defined for power grids. We also showed how these topological benefits can be related to economical aspects of electricity distribution through an improvement in the α and β parameters. The underlying motivation for the present work is to develop decision-support techniques based on CNA metrics to upgrade the power grid to a smart grid and provide methodologies to assess the current infrastructures. In addition, it enables to predict how a change in the topology, according to a certain network model, can be beneficial for the network from an efficiency, resilience and robustness perspective.

From the industrial perspective, where a unique and clear definition of the smart grid term [58] is missing and where the standardization process is at the early stages of development, we consider that the present proposal is useful to make general decisions on how to evolve the grid and (roughly) what costs are entailed. Existing planning techniques will have to be revised in the future, especially for the distribution grid, due to the presence of advanced metering infrastructure (i.e., bidirectional intelligent digital meters at customer location) and distribution automation (i.e., feeders can be monitored, controlled in automated way through two-way communication). In addition, the medium and low voltage grid will no longer be a layer where only energy is consumed, but distributed energy generation facilities (small-scale photovoltaic systems and small-wind turbines) will be attached to this segment of the grid; altogether, these elements are likely to reshape the way planning for medium and low voltage is realized [59] and will also call for new instruments such as the one we propose here. The dynamics of the power system taking place on top of the grid topology is another relevant issue to consider. The existing literature focuses on the high voltage grid by assessing the synchronization properties through a spectral analysis of the graph [60] and how the robustness of networks are influenced by the properties of their spectrum [61]. Even closer to the practical problems is the issue of choosing control parameters and control strategies to apply to the nodes of the networks (generators) to ease the achievement and conservation of synchrony between the generators of a networked power system [62]. We do not consider these last aspects in the present work; however, we envision that the analysis of the dynamics on networks for the electrical distribution grid is an interesting area of research that deserve more precise models to bring the smart grid into the field.

Our future efforts will be devoted to realizing an engineering process that guides the evolution of current network infrastructure towards future topologies optimized for local energy exchange in the smart grid context. Our initial findings suggest that different strategies of adding links to an existing network can be used to improve the distribution grid physical samples [63].

Acknowledgments

The work is supported by the Dutch National Research Council under the NWO Smart Energy Systems programme, contract no. 647.000.004. Pagani is supported by University of Groningen with the Ubbo Emmius Fellowship 2009 and IBM PhD Fellowship 2013-14.

Appendix. Relating topology to economic benefits of electricity distribution

In Section 5 we introduced the association of the grid topology to the cost of electricity. Here we give a thorough explanation of these concepts based on the findings of the work of Pagani and Aiello [10], where they used these metrics to relate topological aspects and electricity cost and applied them to the existing Dutch medium and low voltage infrastructure.

The goal is to consider, from a topological perspective, those aspects and quantities that are critical in contributing to the cost of electricity as elements in the transmission and distribution networks. Economic studies have assessed the components of the cost of electricity. The studies of Harris and Munasinghe [64,65] provide the following aspects that influence price and that are related to topology:

- losses both in line and at transformer stations,
- security and capacity factors,
- line redundancy, and
- power transfer limits.

The topological aspects that we consider provide two sorts of measures: the first one, α, gives an average of the dissipation in the transmission between two nodes

$$\alpha = f(L_{line_N}, L_{substation_N}); \tag{A.1}$$

the second one, β, is a measure of reliability/redundancy on the paths among any two nodes

$$\beta = f(Rob_N, Red_N, Cap_N). \tag{A.2}$$

The functions to explicitly compute the α and β parameters can be expressed as follows:

- Losses on the transmission/distribution line can be expressed by the quotient of the weighted characteristic path length and the average weight of a line (a weighted edge in the graph):

$$L_{line_N} = \frac{WCPL_N}{\overline{w}}. \tag{A.3}$$

- Losses at substation level are expressed as the number of nodes (on average) that are traversed when computing the weighted shortest path between all the nodes in the network:

$$L_{substation_N} = \overline{Nodes_{WCPL_N}}. \tag{A.4}$$

- Robustness is evaluated with a random-removal strategy, and the weighted-node-degree-based removal by computing the average of the order of maximal connected component between the two situations when the 20% of the nodes of the original graph are removed. It can be written as:

$$Rob_N = \frac{|MCC_{Random20\%}| + |MCC_{NodeDegree20\%}|}{2}. \tag{A.5}$$

- Redundancy is evaluated by covering a random sample of the nodes in the network (40% of the nodes whose half represents source nodes and the other half represents destination nodes) and computing, for each source and destination pair, the ten shortest paths of increasing length. If there are fewer than ten paths available, the worst-case path between the two nodes is considered. To have a measure of how these resilient paths have an increment in transportation cost, a normalization with the weighted characteristic path length is performed. We formalize it as:

$$Red_N = \frac{\sum_{i \in Sources, j \in Sinks} SP_{w_{ij}}}{WCPL}. \tag{A.6}$$

- Network capacity is considered as the value of the weighted characteristic path length [10], whose weights are the maximal operating current supported, normalized by the average weight of the edges in the network (average current supported by a line). That is:

$$Cap_N = \frac{WCPL_{currentN}}{\overline{w_{current}}}. \tag{A.7}$$

With these instantiations, Eqs. (A.1) and (A.2) become:

$$\alpha = f(L_{line_N}, L_{substation_N}) = L_{line_N} + L_{substation_N} \tag{A.8}$$

$$\beta = f(Red_N, Rob_N, Cap_N) = \frac{Red_N}{Rob_N \cdot ln(Cap_N)}. \tag{A.9}$$

The aspects considered here are just some of the factors (the ones closely coupled to topology) that influence the overall price of electricity. Naturally, there are other factors that influence the final price, e.g., fuel prices, government policies and taxation, etc., as illustrated for instance in the economic studies of Harris and Munasinghe [64,65].

References

[1] R. Cossent, T. Gómez, P. Frías, Towards a future with large penetration of distributed generation: is the current regulation of electricity distribution ready? Regulatory recommendations under a european perspective, Energy Policy 37 (2009) 1145–1155.
[2] P.L. Joskow, Lessons learned from electricity market liberalization, Energy J. 29 (2008) 9–42.
[3] A.B. Lovins, E.K. Datta, T. Feiler, K.R. Rabago, J.N. Swisher, A. Lehmann, K. Wicker, Small is Profitable: The Hidden Economic Benefits of Making Electrical Resources the Right Size, Rocky Mountain Institute, 2002.
[4] V. Vaitheeswaran, Power to the People, Earthscan, 2005.
[5] G.A. Pagani, M. Aiello, Modeling the last mile of the smart grid, in: Innovative Smart Grid Technologies, ISGT, IEEE PES, 2013, pp. 1–6.
[6] M. Newman, Networks: An Introduction, OUP Oxford, 2010.
[7] R. Cohen, S. Havlin, Complex Networks: Structure, Robustness and Function, Cambridge University Press, 2010.
[8] M. Newman, A. Barabási, D. Watts, The Structure and Dynamics of Networks: Princeton Studies in Complexity, Princeton University Press, 2006.
[9] A.L. Barabási, Linked: The new science of networks, Amer. J. Phys. 71 (2004) 409–410.
[10] G.A. Pagani, M. Aiello, Towards decentralization: a topological investigation of the medium and low voltage grids, IEEE Trans. Smart Grid 2 (2011) 538–547.
[11] G.A. Pagani, M. Aiello, The power grid as a complex network: a survey, Physica A 392 (2013) 2688–2700.
[12] D. Crawford, J. Holt, S.B, A mathematical optimization technique for locating and sizing distribution substations, and deriving their optimal service areas, IEEE Trans. Power Appar. Syst. 94 (1975) 230–235.
[13] G.A. Pagani, M. Aiello, Power Grid Network Evolutions for Local Energy Trading, Technical Report, JBI, University of Groningen, 2012, Available at arXiv:1201.0962.
[14] P. Erdős, A. Rényi, On random graphs. I, Publ. Math. Debrecen 6 (1959) 290–297.
[15] D.J. Watts, S.H. Strogatz, Collective dynamics of 'small-world' networks, Nature 393 (1998) 440–442.
[16] A.L. Barabási, R. Albert, Emergence of scaling in random networks, Science 286 (1999) 509.
[17] H. Jeong, B. Tombor, R. Albert, Z.N. Oltvai, A.L. Barabási, The large-scale organization of metabolic networks, Nature 407 (2000) 4–651.
[18] J. Travers, S. Milgram, An experimental study of the small world problem, Sociometry 32 (1969) 425–443.
[19] W. Aiello, F. Chung, L. Lu, A random graph model for massive graphs, in: Proceedings of the Thirty-Second Annual ACM Symposium on Theory of Computing, STOC'00, 2000, pp. 171–180.
[20] M. Faloutsos, P. Faloutsos, C. Faloutsos, On power-law relationships of the internet topology, in: Proceedings of the Conference on Applications, Technologies, Architectures, and Protocols for Computer Communication, ACM, 1999, p. 262.
[21] V. Colizza, A. Barrat, M. Barthélemy, A. Vespignani, Predictability and epidemic pathways in global outbreaks of infectious diseases: the sars case study, BMC Med. 5 (2007) 34.
[22] M. Boss, H. Elsinger, M. Summer, S. Thurner, The network topology of the interbank market, Quant. Finance 4 (2004) 677–684.
[23] R. Albert, I. Albert, G. Nakarado, Structural vulnerability of the north american power grid, Phys. Rev. E 69 (2004).
[24] D. Chakrabarti, C. Faloutsos, Graph mining: laws, generators, and algorithms, ACM Comput. Surv. (CSUR) 38 (2006) 2.
[25] M.E.J. Newman, The structure and function of complex networks, SIAM Rev. 45 (2003) 167–256.
[26] D.J. Watts, Small Worlds: The Dynamics of Networks between Order and Randomness, Princeton University Press, Princeton, NJ, USA, 2003.
[27] D. Chakrabarti, Y. Zhan, C. Faloutsos, R-MAT: A Recursive Model for Graph Mining, in: Fourth SIAM International Conference on Data Mining.
[28] L.A.N. Amaral, A. Scala, M. Barthélémy, H.E. Stanley, Classes of small-world networks, Proc. Natl. Acad. Sci. USA 97 (2000) 11149–11152.
[29] J. Kleinberg, R. Kumar, P. Raghavan, S. Rajagopalan, A. Tomkins, The web as a graph: measurements, models, and methods, in: Proceedings of the 5th Annual International Conference on Computing and Combinatorics, Springer-Verlag, 1999, pp. 1–17.
[30] J. Leskovec, J. Kleinberg, C. Faloutsos, Graphs over time: densification laws, shrinking diameters and possible explanations, in: Proceedings of the Eleventh ACM SIGKDD International Conference on Knowledge Discovery in Data Mining, KDD '05, ACM, New York, NY, USA, 2005, pp. 177–187.
[31] J. Leskovec, D. Chakrabarti, J. Kleinberg, C. Faloutsos, Z. Ghahramani, Kronecker graphs: an approach to modeling networks, J. Mach. Learn. Res. 11 (2010) 985–1042.
[32] J.K. Gershenson, G.J. Prasad, Y. Zhang, Product modularity: definitions and benefits, J. Eng. Des. 14 (2003) 295–313.
[33] S.K. Ethiraj, D. Levinthal, Modularity and innovation in complex systems, Manag. Sci. 50 (2004) 159–173.
[34] U. Brandes, A faster algorithm for betweenness centrality, J. Math. Sociol. 25 (2001) 163–177.
[35] A. Vázquez, R. Pastor-Satorras, A. Vespignani, Large-scale topological and dynamical properties of the Internet, Phys. Rev. E 65 (2002) 1–12.
[36] R. Cohen, K. Erez, D. Ben-Avraham, S. Havlin, Resilience of the internet to random breakdowns, Phys. Rev. Lett. 85 (2000) 8–4626.
[37] R. Albert, H. Jeong, A.L. Barabási, Error and attack tolerance of complex networks, Nature 406 (2000) 378–382.
[38] L. Garver, Transmission network estimation using linear programming, IEEE Trans. Power Appar. Syst. PAS-89 (1970) 1688–1697.
[39] National Grid, Undergrounding high voltage electricity transmission—the technical issues. Technical Report, National Grid, 2009.
[40] C.W. Anderson, J.R. Santos, Y.Y. Haimes, A risk-based input-output methodology for measuring the effects of the august 2003 northeast blackout, Econom. Syst. Res. 19 (2007) 183–204.
[41] G. Benmouyal, N. Fischer, A. Guzman, J. Mooney, D. Tziouvaras, Advanced transmission line protection system, in: Eighth IEE International Conference on Developments in Power System Protection, Vol. 2, 2004, pp. 445–448.
[42] S.N. Dorogovtsev, J.F.F. Mendes, Evolution of networks, Adv. Phys. 51 (2002) 1079–1187.
[43] S.N. Dorogovtsev, J.F.F. Mendes, Evolution of Networks: From Biological Nets to the Internet and WWW, Oxford Univ. Press, 2003.
[44] D. Liben-Nowell, J. Kleinberg, The link-prediction problem for social networks, J. Am. Soc. Inf. Sci. Technol. 58 (2007) 1019–1031.
[45] A. Barabási, H. Jeong, Z. Néda, E. Ravasz, A. Schubert, T. Vicsek, Evolution of the social network of scientific collaborations, Physica 311 (2002).
[46] R.V. Oliveira, B. Zhang, L. Zhang, Observing the evolution of internet as topology, SIGCOMM Comput. Commun. Rev. 37 (2007) 313–324.
[47] R.V. Solé, M. Rosas-Casals, B. Corominas-Murtra, S. Valverde, Robustness of the European power grids under intentional attack, Phys. Rev. E 77 (2008) 1–7.
[48] P. Crucitti, V. Latora, M. Marchiori, A topological analysis of the italian electric power grid, Physica A 338 (2004) 92–97. Proceedings of the Conference A Nonlinear World: The Real World, 2nd International Conference on Frontier Science.
[49] M. Youssef, C. Scoglio, S. Pahwa, Robustness measure for power grids with respect to cascading failures, in: Proceedings of the 2011 International Workshop on Modeling, Analysis, and Control of Complex Networks, Cnet '11, ITCP, 2011, pp. 45–49.

[50] P. Crucitti, V. Latora, M. Marchiori, Locating critical lines in high-voltage electrical power grids, Fluct. Noise Lett. 5 (2005) L201–L208.
[51] V. Rosato, S. Bologna, F. Tiriticco, Topological properties of high-voltage electrical transmission networks, Electr. Power Syst. Res. 77 (2007) 99–105.
[52] A.J. Holmgren, Using graph models to analyze the vulnerability of electric power networks, Risk Anal. 26 (2006) 955–969.
[53] S. Mei, X. Zhang, M. Cao, Power Grid Complexity, Springer, 2011.
[54] Z. Wang, A. Scaglione, R.J. Thomas, Generating statistically correct random topologies for testing smart grid communication and control networks, IEEE Trans. Smart Grid 1 (2010) 28–39.
[55] Z. Wang, R. Thomas, A. Scaglione, Generating random topology power grids, in: Proceedings of 22 the 41st Annual Hawaii International Conference on System Sciences, p. 183.
[56] A. Scala, S. Pahwa, C. Scoglio, Cascade failures from distributed generation in power grids, arXiv preprint arXiv:1209.3733, 2012.
[57] A. Scala, M. Mureddu, A. Chessa, G. Caldarelli, A. Damiano, Distributed generation and resilience in power grids, arXiv preprint arXiv:1208.5697, 2012.
[58] M.G. Morgan, J. Apt, L.B. Lave, M.D. Ilic, M. Sirbu, J.M. Peha, The many meanings of "Smart Grid". Technical Report, Carnegie Mellon University, 2009.
[59] R. Brown, Impact of smart grid on distribution system design, in: Power and Energy Society General Meeting—Conversion and Delivery of Electrical Energy in the 21st Century, 2008 IEEE, pp. 1 –4.
[60] V. Fioriti, M. Sforna, G. D'Agostino, Spectral analysis of a real power network, Int. J. Crit. Infrastruct. 8 (2012) 354–367.
[61] G. D'Agostino, A. Scala, V. Zlati, G. Caldarelli, Robustness and assortativity for diffusion-like processes in scale-free networks, Europhys. Lett. 97 (2012) 68006.
[62] A.E. Motter, S.A. Myers, M. Anghel, T. Nishikawa, Spontaneous synchrony in power-grid networks, Nat. Phys. 9 (2013) 191–197.
[63] G.A. Pagani, M. Aiello, From the Grid to the Smart Grid, Topologically, Technical Report, JBI, Univ. of Groningen, 2013, Available at arXiv:1305.0458.
[64] C. Harris, Electricity Markets: Pricing, Structures and Economics, Wiley, 2006.
[65] M. Munasinghe, Engineering-economic analysis of electric power systems, Proc. IEEE 72 (1984) 424–461.